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English Language Learner Instruction *(cont.)*

Skills for Math *(cont.)*

Visualizing

Teach students how to visualize so they can create math pictures in their minds. Students use their prior knowledge, known vocabulary, and understanding of math concepts to form mental images for problem solving and other math processes.

Tips for Teaching the Skill

- ★ Ask students what they see in their minds when you read specific math terms.
- ★ Use different kinds of pictures or graphics to help students understand math concepts, such as the characteristics of geometric shapes.
- ★ Translate problems into students' native languages, if possible, to help them draw.
- ★ Use color to help students visualize two- and three-digit operations. Students can use different colors to represent ones, tens, and hundreds as they refine their addition and subtraction skills with regrouping and subtracting across zeros.
- ★ Help students visualize what is happening in a word problem by having them draw pictures to show their understanding of the situation described.



Sample Activity

Have students visualize grids of numbers with the following activity.

1. Provide copies of a grid with numbers one through one hundred (100s chart).
2. Ask students to imagine what pattern they would see if they colored all the multiples of five. Have students color the pattern for multiples of five.
3. Ask them to visualize the pattern they would see if they colored all the multiples of three. Have students use a different color to shade the multiples of three. How close is the colored pattern to what they visualized?
4. Ask students to visualize other patterns and describe them to partners for the partners to color. Partners can then check each other to determine how closely the coloring matched the multiples described.



English Language Learner Instruction *(cont.)*

Teaching Strategies *(cont.)*

Sentence Frames

Use this strategy to provide structure for students learning academic language. Sentence frames—also called sentence starters, sentence stems, or communication guides—have one or more keywords left blank in a sentence. Use sentence frames to do the following:

- ★ help students learn new vocabulary in context.
- ★ introduce complex math concepts.
- ★ help students talk about math.
- ★ help students understand word problems.

Examples: “While reading, I used the strategy to _____”; “My conclusion is _____ because _____.”

Tip for Teaching the Strategy

Create sentence frames based on real-world scenarios.

Sample Activity

Have students create booklets to demonstrate their understanding of prime and composite numbers. Encourage students to write their own sentence frames about prime and composite numbers. Consider sharing the following examples:

- Prime numbers only have _____ (*the number 1*) and _____ (*the number itself*) as factors.
- A prime number only has two _____ (*factors*).
- The numbers 3 and 11 are _____ (*prime*) numbers.
- I know that _____ (*12*) is a composite/prime number because _____ (*I can divide it by more than two numbers*); its factors are 1, 2, 3, 4, 6, and 12.

To make the booklet, have students fold a plain sheet of paper in half lengthwise. Then have them fold it in half width-wise, and width-wise again to make eight sections. Unfold the paper and lay it flat. Fold the paper in half width-wise and cut from the fold just halfway through to the first crease. Open the paper and fold it lengthwise again. Push the end sections together to fold into a little book. Four pages will be formed with two sides each.



Math Language Connections *(cont.)*

Glossary: Math Terms

Math has its own jargon—a vocabulary that can be tricky for struggling ELLs who are still learning the fundamentals of English. ELLs may confuse the word meanings of multiple-meaning words (e.g., *face*, *mean*) or become frustrated when more than one math term is used in a sentence. (e.g., **Predict** which **polygon** will come next in the **pattern**.) To help them overcome these obstacles, copy the following glossary pages for each student in your class. Consider adding the “Glossary: Math Verbs” on pp. 71–72 for additional vocabulary help. Your students will feel more comfortable with this challenging subject if they have a math resource to consult.

acute angle: an angle that is less than 90 degrees



algorithm: a step-by-step solution to a problem

angle: the space between two straight lines or surfaces that touch or cross each other, measured in degrees

area: the amount of space inside the boundary of a flat (two-dimensional) shape, measured in square units

array: an arrangement of objects, pictures, or numbers in columns and rows

associative property of addition: When adding more than two numbers, it does not matter how you group the numbers. (Example: $[3 + 4] + 5 = 3 + [4 + 5]$)

associative property of multiplication: When multiplying more than two numbers, it does not matter how you group the numbers. (Example: $[3 \times 4] \times 5 = 3 \times [4 \times 5]$)

average: the result you get when you add a set of numbers and then divide the sum by how many numbers there are; it’s also called the “mean.” (Example: The average of 2, 4, and 6 is 4, because $2 + 4 + 6 = 12$ and $12 \div 3 = 4$.)

Celsius (C): a measurement of temperature; water boils at 100°C and freezes at 0°C .

centimeter (cm): a measure of length in the metric system, equal to $\frac{1}{100}$ of a meter; $2.54 \text{ cm} = 1 \text{ inch}$

circumference: the distance around the edge of a circle

column: a vertical (up and down) group of objects; this is a column of numbers:

7

11

12

29

commutative property of addition: When adding, you can change the placement of the numbers and still get the same answer. (Example: $2 + 6 = 6 + 2$)



Practical Classroom Applications *(cont.)*

Sample Lesson: Steps to Problem Solving

Objective

Given an introduction/review of problem-solving steps, students will consider possible strategies to solve sample problems.

Vocabulary

eliminate: to leave out or get rid of, as in to eliminate one possibility from among several

problem: a question that needs a solution

solution: an answer to a problem

strategy: a plan of action to achieve a goal

Materials

- ★ “Problem-Solving Strategies” (pg. 97)
- ★ cardstock

Preparation

1. Photocopy “Problem-Solving Strategies” onto cardstock so that each student gets one card. Cut out the cards.
2. Photocopy and enlarge “Problem-Solving Strategies” for classroom display (optional).
3. Prepare sample problem cards, one problem per card (optional for extension activity).

Opening

Introduce the steps of problem solving. Encourage students to compare these steps to any problem-solving steps they have encountered in the textbook or previous lessons. The following steps are based on Polya’s steps to problem solving:

1. Read the problem carefully and make sure you understand what it is asking.
2. Think about the strategies you know. Make a plan for how you will solve the problem. (*Which strategy might be the best to solve this problem?*)
3. Try your plan and solve the problem.
4. Reflect back on your work. (*How well did your strategy work? Does your solution answer the question that the problem is asking?*)

Directions, Part I

1. Write a sample problem on the board. Think aloud and model working through the problem-solving steps. Consider sharing the following example:
 - ★ Jake drew 24 pictures in his sketchbook. Then he colored them using watercolor paints. Jake drew four times as many pictures as Eli. How many pictures did Eli draw? (*6 pictures*)